Asymptotic behavior of pion form factors

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Abstract. We consider the electromagnetic and transition pion form factors. Using dispersion relations we simultaneously describe both the hadronic, time-like region and the asymptotic region of large energy-momentum transfer. For the latter we propose a novel mechanism of Regge fermion exchange.

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Photons interact with quarks, the charged constituents of hadrons and the resulting electromagnetic form factors probe the quark energy-momentum distribution in hadrons. In this article we examine the charged pion electromagnetic form factor $F_{2\pi}(s)$, which is defined by the matrix element $\langle\pi^+(p')\pi^-(p)|J_\mu|0\rangle = e(p'-p)_\mu F_{2\pi}(s)$, and the transition from factor between the neutral pion and a real photon, $F_{\pi\gamma}(s)$ determined by $\langle\pi^0(p')\gamma(\lambda,p)|J_\mu|0\rangle = ie^2/4\pi^2 f_\pi \varepsilon_{\mu\nu\alpha\beta}\varepsilon^{*\nu}(\lambda)p'^\alpha p^\beta F_{\pi\gamma}(s)$. Above, J_μ is the electromagnetic current, $s=(p'+p)^2$ is the four-momentum transfer squared and $f_\pi = 92.4$ MeV is the pion decay constant. Current conservation implies $F_{2\pi}(0)=1$ and, in the chiral limit, axial anomaly determination of the $\pi^0 \to 2\gamma$ decay leads to the expectation $F_{\pi\gamma}(0)\approx 1$. Because at short distances quark/gluon interactions are asymptotically free, it has been postulated that at high energy or momentum transfer, both form factors measure hard scattering of the photon with a small number of the QCD constituents [1, 2, 3]. The available data on the pion electromagnetic form factor ranges up to $|s| \lesssim 10$ GeV [4] and is approximately a factor of three above the asymptotic prediction [5]. Even more spectacular discrepancy is observed in the transition form factor recently measured by BaBar [6]. For momentum transfers as large as $-s \approx 40\mathrm{GeV}^2$ the data suggest that the magnitude of $-sF_{\pi\gamma}(s)$ grows with $|s|$, whereas pQCD predicts $sF_{\pi\gamma}(s) \to 2f_\pi$ as $|s| \to \infty$ [3].

In the following, we relate the form factors in the space-like ($s<0$) and time-like ($s>0$) regions through a dispersion relation (DR), and focus on the dynamics in the asymptotic region, $s \to +\infty$. In view of the BaBar "anomaly" and the apparent failure of the pQCD description, we propose a novel description for the dominant mechanism that drives the asymptotic behavior of the form factors [7].

The discontinuity of $F_{\pi\gamma}(s)$ across the unitary cut is given by

$$\mathrm{Im}\,F_{\pi\gamma} = t^*_{2\pi,\pi\gamma}\rho_{2\pi}F_{2\pi} + t^*_{3\pi,\pi\gamma}\rho_{3\pi}F_{3\pi} + \sum_{X\neq 2\pi,3\pi} t^*_{X,\pi\gamma}\rho_X F_X\,. \tag{1}$$

Here, $t_{X,\pi\gamma}$ (F_X) represent the amplitudes for $X \to \pi^0\gamma$ ($\gamma^* \to X$), respectively and ρ_X is a product of the phase space and kinematical factors (*i.e.* for the 2π intermediate state

$\rho_{2\pi}(s) = s(1-s_{th}/s)^{3/2}/96\pi$). Provided $\mathrm{Im}F_{\pi\gamma}$ vanishes at $s \to \infty$, its real part can be reconstructed for any s from the unsubtracted DR

$$F_{\pi\gamma}(s) = \frac{1}{\pi}\int_{s_{th}} ds' \frac{ImF_{\pi\gamma}(s')}{s'-s}. \tag{2}$$

The two lowest mass intermediate states, $X = 2\pi, 3\pi$ that are dominated by the $\rho(770)$ and $\omega(782)$ resonances, respectively, to a large extent saturate the cut in the hadronic range $s_{th} < s \lesssim 1\mathrm{GeV}^2$. The contribution of a narrow resonance to $F_{\pi\gamma}$ can be well approximated by a Breit-Wigner distribution,

$$F^V_{\pi\gamma}(s) = c^V_{\pi\gamma} m_V^2/[m_V^2 - s - im_V\Gamma_V(s)]. \tag{3}$$

We obtain $c^{(3\pi)}_{\pi\gamma} = c^{\omega}_{\pi\gamma} = 4\pi^2 f_\pi g_{\omega\pi\gamma}/m_\omega g_\omega = 0.493$ (obtained with $\omega \to \pi\gamma$ and $\omega \to e^+e^-$ decay widths yielding $g_{\omega\pi\gamma} = 1.81$ and $g_\omega = 17.1$, respectively), and $c^{(2\pi)}_{\pi\gamma} = c^{\rho}_{\pi\gamma} = 4\pi^2 f_\pi g_{\rho\pi\gamma}/m_\rho g_\rho = 0.613$ (with $\rho \to \pi\gamma$ and $\rho \to e^+e^-$ decay widths leading to $g_{\rho\pi\gamma} = 0.647$ and $g_\rho = 4.96$). The isovector contribution can be further improved using a unitary parametrization of [8]. At higher energies, $s \gtrsim 1GeV^2$ the $K\bar{K}$ inelastic channel and other multi-particle intermediate states are expected to contribute. Unfortunately, since no time-like data are available one cannot unambiguously determine these contributions. A possible determination of the multi-particle hadronic states could be given in terms of quark/gluon intermediate states.

Since the electromagnetic form factor F_X of a composite state decreases with energy-momentum transfer, asymptotically the *r.h.s* of Eq. (1) is dominated by the $X = q\bar{q}$, quark-antiquark intermediate state. Its form factor is $F_{q\bar{q}} = 1$, and the state contributes to $\mathrm{Im}F_{\pi\gamma}$ via the $q\bar{q} \to \pi\gamma$, P-wave scattering amplitude, $t_{q\bar{q},\pi\gamma}$ as illustrated by the last diagram in Fig.1a. The $q\bar{q}$ contribution shown in Fig.1a may be compared to the one in Fig.1b, which represents the asymptotic contribution as predicted by pQCD. In the latter, the $q\bar{q} \to \pi\gamma$ scattering amplitude, shown to the right of the vertical cut line, is given by a free quark propagator exchanged between the final state pion and photon. In the kinematically relevant domain $s \gg t$, t being the momentum squared carried by the exchanged quark, the amplitude $t_{q\bar{q},\pi\gamma}$ is expected to have a Regge behavior [9] $t_{q\bar{q},\pi\gamma}(s,t) = \beta_\pi(t)\beta_\gamma(t)s^{\alpha_q(t)} \approx e^{bt}s^{\alpha_q}$, where β_π, β_γ are residues of the exchanged quark at the corresponding vertex. The difference between the free, Fig.1b and the

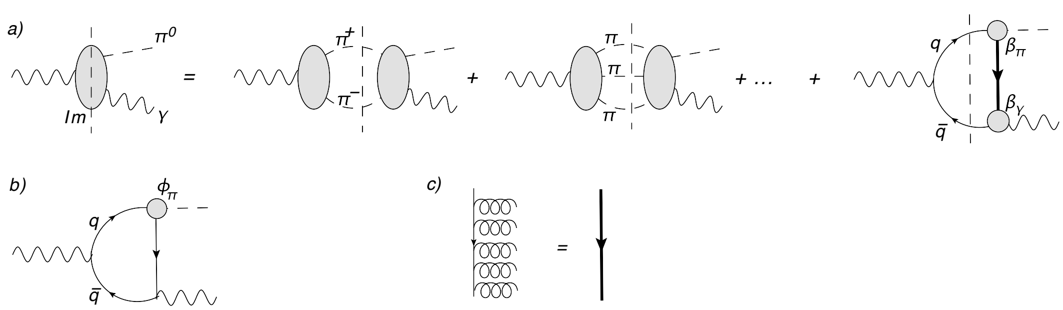

FIGURE 1. Hadronic and asymptotic contributions to the π^0 transition form factor.

Regge propagator Fig.1a can originate from the sum of ladder gluons in the wee region

(*cf.* Fig.1c). The quark Regge trajectory $\alpha_q(t) \approx \alpha_q(0) + \alpha'_q t$ is not known; however, phenomenologically it can be related to the leading Regge exchange in $\pi\pi$ scattering. Identifying reggeized ρ and f_2-exchanges with reggeized $q\bar{q}$-exchanges then leads to the expectation $A_{q\bar{q}} \sim s^{2\alpha_q(t)-1}$, or $\alpha_q(t) \sim \frac{1}{2}(\alpha_\rho(t)+1) \approx 0.75 + 0.45t/GeV^2$. Further analysis of phenomenological implications of this quark reggeization will be given in the forthcoming paper [10]. After projecting onto the p-wave, the energy dependence of the asymptotic, $q\bar{q}$ contribution to $ImF_{\gamma\pi}$ is therefore expected to behave as

$$\mathrm{Im} F^{(q\bar{q})}_{\gamma^*\pi\gamma}(s) \to c^{(q\bar{q})}_{2\pi}(s/GeV^2)^{\alpha_q(0)-3/2}. \tag{4}$$

Combining the ω and the ρ resonance contributions of Eq. (3) with the asymptotic form of Eq. (4) and making the simplifying assumption that the first two contribute to $ImF_{\pi\gamma}$ for $s < 1$ GeV^2 while the asymptotic part saturates $ImF_{\pi\gamma}$ for $s > \mu^2$, we fit the available data using Eq. (2) with the single free parameter $c^{(q\bar{q})}_{\pi\gamma}$ that determines the normalization of the asymptotic contribution. The result is shown in the left panel of Fig.2. It is worth noting that even at largest values of $-s$ the bare $q\bar{q}$ production gives only about 50% (dash-dotted line in the left panel of Fig. 2) of the form factor with the remaining half coming from the resonances.

In the case of the pion electromagnetic form factor, the discontinuity reads

$$\mathrm{Im} F_{2\pi} = t^*_{2\pi,2\pi}\rho_{2\pi}F_{2\pi} + t^*_{K\bar{K},2\pi}\rho_{2K}F_K + \sum_X t^*_{X,2\pi}\rho_X F_X \tag{5}$$

where in the sum is over intermediate states ($X \neq 2\pi, K\bar{K}$) in $\gamma^* \to X \to 2\pi$ and where we separated the two channels $X = 2\pi$ and $X = K\bar{K}$ which phenomenologically are most significant in the hadronic domain. Above the inelastic threshold, $s > s_i$, the unitarity relation now involves both $ImF_{2\pi}$ and $ReF_{2\pi}$ and can be solved algebraically. Assuming that the elastic amplitude, $t_{2\pi,2\pi}$ asymptotically approaches the diffractive limit, $t_{2\pi,2\pi} \to i/2\rho_{2\pi}$, from Eq. (5) one finds

$$F_{2\pi}(s) \to 2i \sum_{X\neq 2\pi} t_{X,2\pi}\rho_X F^*_X \to 2it_{q\bar{q},2\pi} \propto is^{\alpha_q(0)-3/2}. \tag{6}$$

Except for the expected energy dependence, we do not know separately the real and imaginary parts of $t_{q\bar{q},2\pi}$. Assuming, as in the case of the transition form factor, that the real part of the discontinuity due to $q\bar{q}$ state has the energy dependence given by the reggized quark exchange, we can compute $F_{2\pi}$ using Eq. (5) and the Cauchy representation. We approximate the sum over inelastic channels by the single $K\bar{K}$ channel, and above $s \geq \mu^2$ the residual $q\bar{q}$ continuum with

$$\mathrm{Re}\, t^*_{q\bar{q},2\pi}\rho_X = c^{(q\bar{q})}_{2\pi}(s/GeV^2)^{\alpha_q(0)-3/2}. \tag{7}$$

For the $t_{2\pi,2\pi}$ and $t_{K\bar{K},2\pi}$ amplitudes we use the parametrization from [11]. We parametrize the isovector kaon form factor F_K using Breit-Wigner distributions which include the $\rho(770)$, $\rho'(1400)$ and $\rho''(1700)$ [12]. Finally we fit the available data on $|F_{2\pi}(s)|^2$ with five parameters: the magnitude and phase of the ρ' and ρ'' contributions

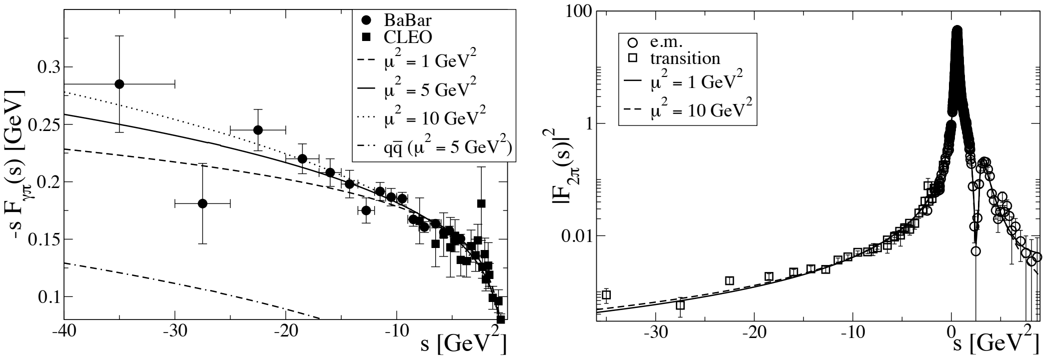

FIGURE 2. Left panel: our results for $|F_{\pi\gamma}(s)|$ in the space-like region for $\mu^2 = 1$ GeV^2 (dashed line), 5 GeV^2 (solid line), 10 GeV^2 (dotted line), in comparison with the experimental data from [6, 13]. The Regge contribution with $\mu^2 = 5$ GeV^2 is shown separately (dash-dotted line). Right panel: our results for the pion electromagnetic form factor for $\mu^2 = 1$ GeV^2 (solid line) and $\mu^2 = 10$ GeV^2 (dashed line) vs. experimental data on the time-like and space-like e.-m. form factor from [4] (solid circles).

to F_K and $c_{2\pi}^{(q\bar{q})}$ –the magnitude of the $q\bar{q}$ continuum, Eq. (7). In the right panel of Fig. 2, we display our results for the electromagnetic pion form factor F_π in the range -40 GeV$^2 \leq s \leq 10$ GeV^2. We confront them with the available experimental data for the electromagnetic form factor for -10 GeV$^2 \leq s \leq 10$ GeV^2 and the transition form factor for -40 GeV$^2 \leq s \leq -0.8$ GeV^2 (both are normalized to 1 at $s = 0$). First, we note that in the space-like region the data sets for the two form factors look identical. One can see that our model describes all the available data throughout the shown kinematics. In the case of the electromagnetic form factor, our result is a prediction for the s-dependence at large $|s|$, where no data exist so far. In particular, we predict that, as for the transition form factor case, $|sF_{2\pi}(s)|$ has to rise asymptotically roughly as $s^{1/4}$, unlike pQCD predictions that feature at most a logarithmic limit for that combination.

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REFERENCES

1. A. V. Efremov and A. V. Radyushkin, Theor. Math. Phys. **42**, 97 (1980)
2. A. Duncan, A.H. Mueller, Phys. Rev. D **21**, 1636 (1980).
3. G.P. Lepage and S.J. Brodsky, Phys. Lett. B **87**, 359 (1979); *ibid.* Phys. Rev. D **22**, 2157 (1980).
4. B. Aubert *et al.*, Phys. Rev. Lett. **103**, 231801 (2009); see Ref. [7] for the complete list of references.
5. S.J. Brodsky, G.R. Farrar, Phys. Rev. Lett. **31**, 1153 (1973).
6. B. Aubert *et al.*, Phys. Rev. D **80**, 052002 (2009).
7. M. Gorchtein, P. Guo, A. P. Szczepaniak, arXiv: 1102.5558.
8. T. N. Pham, T. N. Truong, Phys. Rev. D **14**, 185 (1976); T. N. Truong, Phys. Rev. D **65**, 056004 (2002).
9. V. S. Fadin, V. E. Sherman, Sov. Phys. JETP **45**, 861 (1977).
10. M. Gorchtein, P. Guo, A. P. Szczepaniak, *in preparation*.
11. P. Guo, R. Mitchell, A.P. Szczepaniak, Phys. Rev. D**82**, 094002 (2010).
12. F. Felicetti, Y. Srivastava, Phys. Lett. B **107**, 227 (1981).
13. J. Gronberg *et al.*, Phys. Rev. D **57**, 33 (1998).